A Downsian model of long standing legislative majorities.

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1 Longstanding Legistlative Majorities

The fundamental question for students of elections is why persons or groups get elected. A fast-following question is why they keep getting elected, if they so do. This latter question has seen a much smaller amount of attention. This is particularly the case in the formal literature. Many models predict equilibria where the two major contestants have equal probabilities of winning elections and the matter is resolved by the flip of a coin; it makes little sense to contemplate a repeated winner.

In the context of legislatures, however, examples of dominant parties, those which seem habitually to be the majority party, spring readily. The Democratic party in the United States was a long standing example, as was the Liberal Democratic party in Japan. When writing on the elections in a single country, authors often deal with these as exceptions, explaining why their country does not conform to the back and forth struggle we expect, but instead some party has a longstanding historical dominance in the legislature[1].

Looking across countries, however, we begin to see these cases are not the exception. Indeed, for example, across all countries with single member districts and free and fair elections for at least twenty years, the party which has held the legislative branch for the most time (since World War II or independence), has been the majority party on average over seventy percent of the time. That is, in each country, on average, some party has held power in the legislative branch *more than twice the time of all other parties combined* (individual country data is given in table 1). This result is unfailing whether one breaks down countries by economic status, population over or under a million, former colonies or former colonial powers.

This paper explores one contributing causal explanation to this phenomenon. In a simple model of parties as collections of incumbents, it is shown that the distribution of Median voters across districts can help or harm parties in equilibrium. In equilibrium in asymmetric distributions the members of one party will rationally choose a policy position which in expectation gives their party a strict minority of seats.

The model is conceptually simple but computationally challenging. The implications of this model are empirically tested using a cross section of state legislatures. In so doing, the theory section demonstrates some of the advantages of rigorous methods of simulation with analytically challenging

[1]For example, an entire edited volume *"Uncommon Democracies: The One-Party Dominant Regimes"* is devoted to explaining the special cases of a large number of countries. The obvious rejoinder is that if you can fill an entire book with such examples, dominant party systems are not "uncommon."

Country	Majority
Bahamas	.78
Barbados	.75
Botswana	1
Belize	.59
Canada	.68
Dominica	.70
Grenada	.73
Jamaica	.72
New Zealand	.69
St. Kitts and Nevis	.58
St. Lucia	.60
St. Vincent	.75
Trinidad	.60
United Kingdom	.65
United States	.82

Table 1: Countries with single member districts and the fraction of the years since independence or World War II that the dominant party has had a majority in the legislature.

problems. We employ a method of Monte Carlo integration so as to turn a highly non-linear non-parametric model into a model with linearly testable predictions. This technique is of potential use in many situations of model testing. Also I present a method of ideal point revelation for median voters using hierarchies of datasets that avoids many crippling problems with common proxies such as presidential vote share.

2 Downs and Divergence

Black's median voter theorem and the Downsian model of elections hold that in a two candidate race over a single policy dimension, both candidates should converge on the position of the median voter. This is a powerful result, especially as the logic seems so instinctively correct. Once grasped it seems obvious, and faced with such obviousness it becomes upsetting when candidates disobey our rules.

The Downsian logic is easily applied to two party elections over a set of single member districts. By Black's median voter theorem we know that whichever national platform is closest to the median in a particular district will carry that district. When a party wins a district median voter they also win that district. Thus we can abstract by one degree and consider an election between two parties over all the district median voters. Again, in this Downsian world, with parties seeking to gain the majority, the parties will converge to the median of the district medians voters. Again we have a strong and intuitive convergence result. And yet instead we see parties that take great pains to diverge across a number of issues. It often seems that in drawing up platforms, internal consistency is entirely sacrificed so as to hold positions contrary to the opponent party.

There have been many attempts to explain the divergence that is seen in actual elections of two candidates. Each of them relax an assumption, or generally a series of assumptions in the Downsian model such as credibility (Alesina 1988), simple majority rule (Cox 1990, Myerson 1993) non-abstention (Hinich et al. 1972) perfect information (Calvert 1985) obstructed entry (Palfrey 1984) or candidate indifference over policies (Wittman 19**, Aldrich 1983). Probabilistic voting models can sometimes point to divergence also. However, while some of these offer convincing argument for why individuals would want to diverge in single races, none of these (with the exception to some extent of Aldrich's (1983, 1995) ambition theory) offer suggestions for why parties collectively would wish to diverge in their platforms[2]. This paper will focus on divergence between party platforms in single member districts.

One recent model that attempts to tackle this has been proposed by Ansolabehere and Snyder (1996) building on the work of Snyder (1994). Their model uses the Downsian framework, retaining its assumptions, in particular assuming that these elections are based on national party platforms that can be described in one dimension[3]. Also they allow that information about these platforms is near perfect, and moreover, that these platforms are credible[4] and voters believe parties can enact them if they attain the majority. Instead of weakening an assumption, they add a complication

[2]Palfrey (1984) also presents a model of party divergence, where parties act as unitary actors to position themselves to prevent entry. It can be shown that if we create a scale of how much parties value preventing entry versus maximizing the probability of attaining a majority, then this will create a model with the exact same dynamics as the model presented, and this scale will map directly onto the weights in the candidate utilities of the Snyder and Ansolabehere model

[3]While single dimension models are generally chastised, and their results never translate to higher dimensions, empirical analyses of party positions, and public perceptions of party positions argue that this is a reasonable approximation.

[4]Indeed, credibility may be interpreted as falling out of their model, and not an assumption, as it will be seen that the policy platforms adopted are the ones the party would most like to establish. They are incentive compatible.

by introducing the Downsian model to the theme of Mayhew (1974) in that parties are made up of individuals, and these individuals want to be reelected. While the elected officials of some party in the legislature may jointly all wish to be in the majority party they also have private interests, namely reelection within their own particular district. In their model, legislator i in party j has an expected utility of the form:

$$U_{ij} = W_1 Prob(\text{candidate } i \text{ wins a seat}) + W_2 Prob(\text{party } j \text{ wins the majority}) \tag{1}$$

That is, there is some utility to winning a seat, even if the party does not win the majority, and some utility to a legislator who loses his seat, but whose party gains the majority[5]. Furthermore, by this specification these utilities are separable. The 'standard Downsian model' is a special case of their model where $W_1 = 0$ and $W_2 = 1$. However, it does seem implausible to believe party members care nothing about their own personal reelection. As Mayhew quotes "All members of Congress have a primary interest in getting reelected. Some members have no other interest." It is these latter members we shall initially consider.

3 Introduction to the Model

Following their model, consider that voters have differential utility between candidates from parties L and R of the form:

$$\begin{aligned} \delta U &= v_L + u_L(L - z) - v_R + u_R(R - z) && (2) \\ &= v + u_L(L - z) + v_R(R - z) && (3) \end{aligned}$$

where $u_C(C - z)$ is the utility a voter has for candidate C's platform position relative to their own ideal point z, and v_C is the utility the voter has for their position on some "valence issue" or "valence score" distributed probabilistically and chosen by nature after the party chooses their platform positions. A valance issue might be fluctuations in the economy about the time of the election or a scandal or media receptiveness/hostility.

Consider the case of legislators who care simply about reelection to their seat (in the terminology of the paper "office-seeking"). They want the party platform to be set as close as possible to the median voter of their district. Given (3) this maximizes the probability attached to W_1 and since W_2 must

[5] *To the victors go the spoils:* since the majority party has a slew of positions and patronage at its disposal, it is not inconceivable that a legislator can gain something form his party winning the majority, even if they lose their seat in the election.

be zero for such legislators, this likewise maximizes the legislator's individual utility. However, each legislator in the party wants the platform at the median *of their particular district*. If there is a vote among the party on the position of the platform, then the platform will fall to the position of the median incumbent in the party[6]. Quite intuitively one can see that this leads to divergence in the platforms parties will adopt[7]. How exactly this will fall out over time and repeated interaction between the parties is not obvious. Nor is it computationally simplistic.

The purpose of this paper is to push this model a little more and derive some of its ramifications beyond platform divergence, absent any additional assumptions. We expand its principles to more interesting settings, and develop an intuition about the degree and nature of the divergence such a model would impose. In particular, we will be interested in its predictions about the extent or existence of long standing legislative majorities.

4 Convergent Equilibria in the Policy Space

Assume then that there exists a Left and Right party, adopting platforms L and R in some single dimension. Then there exists a cutpoint, C, defined by:

$$C = \frac{L + R}{2} + v \tag{4}$$

where v is the common valance shock. Voters to the right of C vote for the Right party, and voters to the left vote for the Left party. Districts whose median voter fall to the right of C elect candidates from the Right party, and vice-versa. If the median of all the district median voters sits to the right of C, the Right party obtains a majority in the Legislature. The obvious question is where should we expect C over time?

Assume $v = 0$. Given C_{t-1}, we can predict where the next party platforms L_t and R_t will reside. We know L_t is the median of the Left party districts,

[6]Here the model is assuming that only the incumbent legislators in the party are the ones voting on the party platform, and their votes/resources have equal weight.

[7]This divergence in platform can also be credible to the electorate, if also before the election the party first votes on a party leader, and then votes on how much power to give him, assuming he can not be constrained at a later point. It will be in equilibrium to choose the member closest to the position of the platform (the median member of the party in the above case where $W_1 = 1$) and give him powers to enforce his wishes. When the legislative session begins the majority party leader will get his ideal point making the platform proposed in the election credible.

so:

$$\frac{\int_{-\infty}^{L_t} D(y)dy}{\int_{-\infty}^{C_{t-1}} D(y)dy} = \frac{1}{2} \tag{5}$$

where D(y) is the distribution function of the district median voters. Some cutpoint ,C', is then an equilibrium if:

$$L_t(C', D(y)) = L_{t-1} \text{ and } R_t(C', D(y)) = R_{t-1} \tag{6}$$

since

$$C_t = \frac{L_t + R_t}{2} = \frac{L_{t-1} + R_{t-1}}{2} = C_{t-1} \tag{7}$$

In deriving this equilibrium cutpoint, we have assumed $v = 0$, that is the absence of valance shocks. However, this equilibrium cutpoint,C' is convergent for $v > 0$ and symmetric, that is:

$$\mathrm{E}[\ |C_t - C'|\ ] < \mathrm{E}[\ |C_{t-1} - C'|\ ] \tag{8}$$

Thus, when we have valance shocks, and one moves the cut point away from the equilibrium in some period, we should expect each subsequent cut point to move back towards the equilibrium.

5 Elections in Symmetric and Asymmetric Districts

Consider a uniform distribution of district median voters over the zero to one interval. We can rewrite (5) as:

$$\frac{\int_{0}^{L_t} dy}{\int_{0}^{C_{t-1}} dy} = \frac{1}{2} \qquad \frac{\int_{R_t}^{1} dy}{\int_{C_{t-1}}^{1} dy} = \frac{1}{2} \tag{9}$$

and solve to find:

$$L_t = \frac{C_{t-1}}{2} \qquad R_t = \frac{C_{t-1} + 1}{2} \tag{10}$$

$$C_t = \frac{L_t + R_t}{2} = \frac{2C_{t-1} + 1}{4} \tag{11}$$

from which our condition in (7) gives us:

$$C' = \frac{2C' + 1}{4} \qquad \Rightarrow \qquad C' = \frac{1}{2} \tag{12}$$

Thus for a uniform distribution of district medians, the equilibrium cutpoint falls at the median of the distribution, M. The equilibrium is then for the parties to tend towards splitting the districts evenly. Indeed, we can easily see this will be true for all symmetric distributions of district median voters. Since C' is equidistant from the equilibrium party platforms, L' and R', then $C' = M$, iff $|L' - M| = |R' - M|$ which will be true if:

$$\int_{-\infty}^{M-A} D(y)dy = \int_{M+A}^{\infty} D(y)dy, \quad \forall A \tag{13}$$

For symmetric D(y), this will always be true, thus $C' = M$, and the parties tend to split the districts evenly. However, this need not true for asymmetric distributions. Consider the distribution $D(y) = by$ over the same zero to one interval. Solving (5) gives us:

$$L_t = \frac{C_{t-1}}{\sqrt{2}} \qquad R_t = \sqrt{\frac{C_{t-1}^2 + 1}{2}} \tag{14}$$

Which given our equilibrium condition (7) produces:

$$C' = \frac{1}{2\sqrt{2 - \sqrt{2}}} \tag{15}$$

However, the median of the distribution is $M = \frac{1}{\sqrt{2}}$. As we see in figure 1 this puts the cutpoint to the left of the median of the district medians, and in an average election, R should expect a majority of the seats. Party L may occasionally win given a very favorable valance shock, but will not retain the majority for as long as R will, on average.

6 Elections in Bimodally Distributed Districts

A popular contention in American politics is that these district medians are bimodally distributed (Fiorina 1989). Consider a distribution of districts composed of two overlapping univariate normal distributions of the form:

$$D(y) = S\ \mathbf{N}(y|\mu_1, \sigma_1^2) + (1 - S)\ \mathbf{N}(y|\mu_2, \sigma_2^2) \tag{16}$$

where $0 < S < 1$. In the special case of $\mu_1/\sigma_1 \ll \mu_2/\sigma_2$, that is when the modes do not overlap much, then:

$$L' = \mu_1, \quad R' = \mu_2 \tag{17}$$

$$C' = \frac{\mu_1 + \mu_2}{2} \tag{18}$$

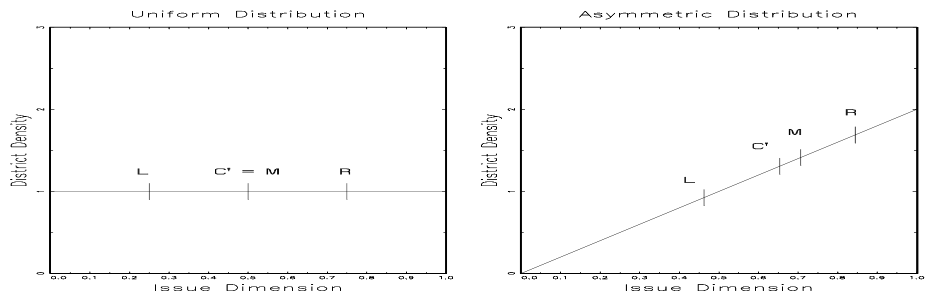

Figure 1: *Graphs showing the positions of the equilibrium platforms of the Left and Right parties, the resulting equilibrium Cutpoint and the location of the Median of the district median voters, for simple symmetric and asymmetric distributions of districts. Note that in the asymmetric distribution the Right party captures the Median of the medians in equilibrium, and thus gains a majority.*

Thus, here C' is not at the position of the median of the district medians, rather, in equilibrium we expect the Left party to win in S of the districts, and the Right party to receive $(1 - S)$ of the seat share in the legislature. If $S > .5$ then the Left party can expect a majority, and vice-versa.

This is a very specialized case of (16) though. However, there is not a closed form solution for the cumulative normal, and Taylor approximations become intractable fast. This make this a good problem for Monte Carlo integration. This also allows us to add in a non-negligible stochastic v term. The stochastic term make the model dynamic. For $v = 0$, C' is the static (and attractive) equilibrium. For $v > 0$ there will be some movement about this equilibrium, and the position of C in any round will be a function of the distribution, $D(y)$, the previous cutpoint and the shock. I allow the series to run for a very long time and determine the characteristics of the series.

In figure 2 we constrain the variances of the two normal distributions to be equal, but sixty percent of the probability mass is in the left peak. Clockwise, the first graph shows the distribution of the district medians. The solid line represents the average location of the cutpoint, and the dashed lines its ninety percent confidence region. We see that on average, the parties each capture all the districts in one of the modes and the cutpoint falls in the saddle between them. The next graph shows the probability of the seat share for the Left party. We see that eighty-six percent of the time the Left party wins

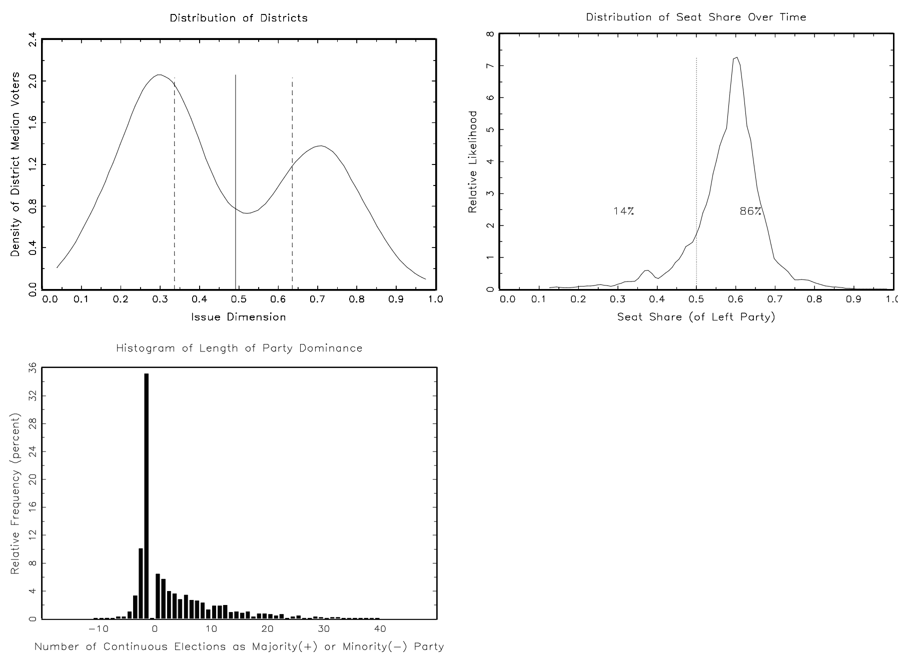

Figure 2: ***Unequally weighted modes:*** *Given asymmetric size of the modes of the distribution, one party, here the Left party, is more likely to capture a majority in any election, and maintains that majority longer.*

a majority of the seats, and the most likely seat share seems to be about sixty percent. The probability distribution in this graph can not tell us the expected number of elections the Left party should sequentially maintain its majority, since sequential draws are not independent. For that we turn to the last graph. We see that the average length of elections where the left party is continuously the majority party is between eight or nine, while the right party's average length of term as majority party is just one and a half elections, and has negligible probability beyond four.

A similar result is found in figure 3. Here the modes are now of equal weight ($S = .5$) but the variance of the right mode is tighter than the variance of the left mode ($\mu_1 > \mu_2$). Again the distribution is asymmetric. Here the Right party becomes the dominant party winning a majority of seats seventy-three percent of the time, and enjoying longer terms as the majority party.

7 Conclusion

The assumptions of the Downsian world can help us to understand the causes of long standing legislative majorities, even though on its face, the Downsian logic would deny the existence of what we are trying to explain. Asymmetry in the underlying distribution of districts can lead incumbents to collectively create policy platforms that seriously harm their party and make it a permanent minority. However, this is individually rational behavior on the part of each of the legislators concerned, and even collectively they would not want some one to force them to do otherwise. The theory can be shown to predict that increasing asymmetry in the distribution of the median voters of districts leads to increasing stable long standing majority (and hence minority) parties. The more incumbents value reelection, the more this is likely also.

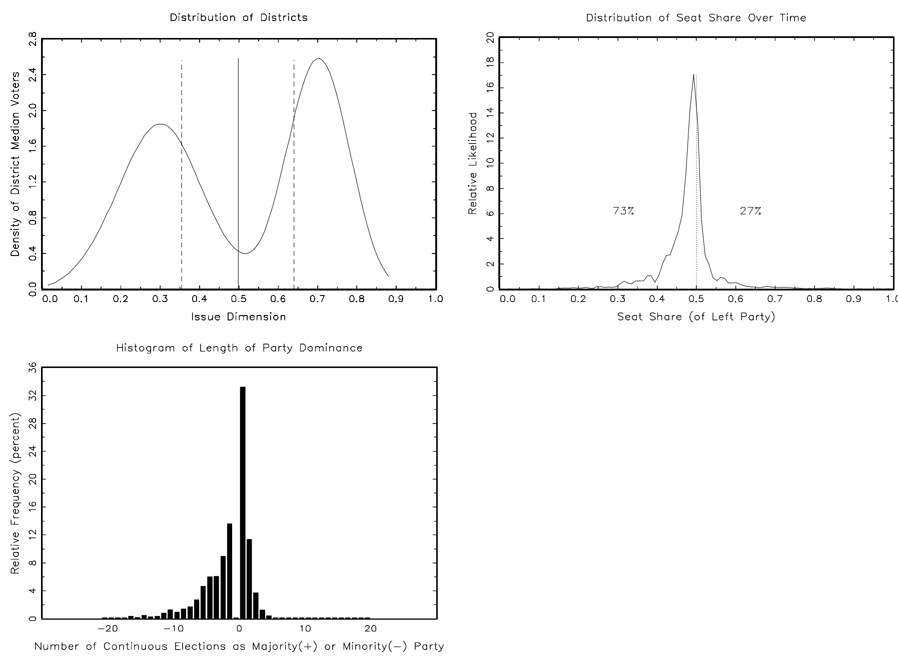

Figure 3: ***Modes with Unequal Variance:*** *Similar to before, even when the modes have the same weight, simply changing the variance of each mode allows one party, here the Right party, to enjoy longer terms of consecutive elections as the majority party.*

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