Redistribution of Voteshares

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ABSTRACT

We detail a model of compositional data for reallocating voteshares under counterfactual scenarios. This builds on a system of models Aitchison (1986), Katz and King (1999) and Honaker, Katz and King (2001), that are used when the inference can be seen as a missing data problem with auxiliary information about bounds. By investigating a series of *undervoting* problems in 2006 with electronic voting, where unexpectedly large numbers of voters did not vote because of ballot technology problems, we are able to test this model in a series of similar races. Because we also have collected individual level "ballot image" data, this gives us a number of reasonably comparable applied settings to judge the average performance of these compositional models and measure the information loss from using aggregated voting returns.

1. INTRODUCTION

It is often the case in quantitative work, that the simplicity with which one can reach an answer does not depend on the simplicity of the question but rather the quality of the available data. Reallocation of voteshare data is generally a perplexing statistical issue even when the questions to which these methods are addressed seem clear cut.

Counterfactual questions of broad interest abound in recent elections. How would the 2000 Presidential election in Florida have turned out if Nader had not been on the ballot (Herron and Lewis, 2007), or if the butterfly ballot had not been used (Wand et al. 2001) or late postmarked overseas ballots not been counted (Imai and King, 2004). How would the 2004 Presidential election in Ohio have turned out if all districts used the same ballot technology? In other party systems common questions might be how votes would be redistributed if certain regional or minority parties dropped out of parliamentary races, or if transferable voting rules were adopted.

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Such questions are often hard to address because the only readily available data to answer them are aggregated voting returns. Therefore the key variables are *compositional* in nature; they are fractions, split between candidates or ballot choices, which must sum up to one-hundred percent. Sometimes, an additional problem is that the nature of the composition changes between the observed data and the counterfactual estimate. That is, when we redistribute votes from one choice to the other possible choices, the dimension of the problem, and all the resulting transformations may change.

We investigate one method of reallocation of aggregated voteshare data, building on the models of Aitchison (1986), Katz and King (1999) and Honaker, Katz and King (2001). We compare the ability of such compositional models in aggregated data to the ideal solutions possible when individual level "ballot image" data is available. We are empirically fortunate, from the vantage of this project, that the 2006 elections in Florida suffered from a number of similar undervoting problems, across different races and different counties, which gives us a number of reasonably comparable applied settings to judge the average performance of these models. We model the reallocation question of what would have happened in each undervoting situation if that ballot race in that county had used the technology of a neighboring county (that did not create increased undervoting), and judge and compare the resultant information loss when one is forced to use a compositional model on aggregated returns, compared to the ideal situation of individual ballot images.

2. BACKGROUND UNDERVOTE PROBLEMS IN 2006

The 2006 elections ushered in the widespread use of electronic voting machines, and with them the promise of fairer elections, more accurate to the intentions of voters. While technology continues to offer that promise, several of the problems with conventional ballots appear to have electronic equivalents, and the accumulated knowledge and study of ballot design is still relevant to current voter technology. One emergent problem in 2006 was unexpectedly large quantities of undervoting, that is, failure to register a vote for a candidate in a given race. While some undervoting is always intentional abstaining on the part of the voter, the quantity of undervoting in certain elections pointed to additional causes. Most notably, in Florida's 13th Congressional District, Democrat Christine Jennings narrowly lost to Republican Vern Jennings by 369 votes, while a disproportionate 17,763 electronic ballots (or almost fifteen percent) registered no vote for Jennings in Sarasota county. This number was roughly five times the rate of undervoting in the other counties[1] in this Congressional district, and crucially, was a county in which Jennings ran very strongly, leading her opponent Buchanan by seven percent. Other instances of undervoting occurred in other races in other counties in Florida, such as the Attorney General race in Charlotte and Lee counties, and the Chief Financial Officer race in Collier county, among others. Frisina, Herron, Honaker and Lewis (2008) demonstrate evidence that undervoting occurred across races in Florida because of ballot formatting (although see also Mebane, 2008). Each county uses its own ballot

[1] The undervoting rate in the 13th Congressional race was 2.6 percent outside Sarasota, and 14.9 percent inside Saratoga.

technology and designs its own ballot layout. Where pages of the electronic ballot grouped together multiple races, this resulted in some races being neglected by voters, likely because they failed to see or look for the race as they moved through the ballot screens.

Frisina et al. conduct a number of analyses to try to estimate the counterfactual of how the election would have turned out if Sarasota had used the same ballot format as the other counties in that Congressional district. One analysis uses the certified aggregated precinct returns, the level of data most typically or most readily available to voting researchers. A further analysis used individual "ballot image" data in a parametric logistical model to reallocate Sarasota undervoters, while a third analysis used a nonparametric matching approach with this same data. Each of their analyses concludes that Jennings would have won this election instead of Buchanan in this counterfactual situation where Sarasota avoided rampant undervoting by corrected ballot design.

In this paper we detail the estimation method developed by Frisina et al., for reallocation with compositional data, conceptualized as a missing data problem with auxiliary bounding information. We run this model across a set of electoral races that all suffered similar undervoting problems in the 2006 general election. Comparing these results to those obtained with the ideal, individual level "ballot image" data, we can judge performance and measure the information loss resulting from the aggregation of vote returns[2]. We are interested in running these comparisons in a number of Florida races, however, in what follows we set out the logic of the model using Florida's 13th Congressional race as it is the case we have explored in most depth in earlier work (Frisina et al. 2008) and has received the most popular media attention with regards to undervoting and electronic ballots.

3. ALLOCATING UNDERVOTES TO IN FLORIDA'S 13TH CONGRESSIONAL RACE

In Sarasota county, a total of 120,686 touch screen ballots were cast in either early or election day voting. Of these, 17,811, or 14.8 percent, did not vote in the thirteenth district Congressional election, while only 2.6 percent undervoted across other counties in this same race. If we assume that a similar 2.6 percent of voters in Sarasota would have intended to undervote, then that means roughly 14,000 voters in Sarasota county would have cast a vote in this race if the touch screen ballot had been the same as in other counties.

There are two stylized stories we might tell to explain why these 14,000 voters did not cast a vote in this election. This surplus level of undervoting might be driven entirely by some voters accidentally not seeing the election, or it might be driven by indifference on the part of certain types of voters. For a simple terminology, let us define any ballot which did not choose a candidate as an *undervote*. Some of these undervotes were caused by individuals who saw

[2] Although we trust the compositional model developed, it is one of the goals of this paper to try to make this model redundant by demonstrating the information loss in aggregated returns, and thus encouraging the timely and widespread release of "ballot image" data, particularly given the increasing work and interest in "election forensics," as a term coined by Mebane.

the election and chose not to vote, while the rest were caused by people who did not initially see the choice because of the ballot design, but would have seen the choice if given the ballot used in other counties. Call these ballots the *suppressed* votes[3], and the votes which would have been cast from the suppressed ballots the *intended* votes.

3.1. *Suppression by Random Accident*

At one extreme, we might suppose that the rate of suppressed votes is explained entirely by the mechanics of the ballot. The touch screen ballot design in Sarasota county led to a certain probability of "accidents" where voters did not see the election and did not ever face the choice they would otherwise have made between voting for the Republican or Democratic candidate, or indeed choosing not to cast a ballot. If the surplus undervoting is driven by completely random accidents, then the suppressed voters are a completely random sample of the voters in each precinct. From this assumption, two points can be made. First the distribution of intended votes should follow (within the rules of sampling) the distribution of votes cast by the voters who were not suppressed. If this model were true, then the roughly 14,000 intended votes would have fallen in about the same split as the unsuppressed votes in Sarasota county, and split roughly 7,500 for Jennings and 6,500 for Buchanan. This would result in Jennings picking up 1000 votes over Buchanan and overcoming the 369 vote deficit on the day, thus Jennings would win the election. If this simple scenario is true, then we have a very simple root to an answer, we do not need precinct level data, let alone individual level data, and we can make our counterfactual estimate based simply on the county-aggregated election returns.

Second, both candidates should have received fewer votes than they would have received if the intended votes had been cast, as suppression transfered votes from their columns to the undervote column. This second point can be clearly shown. If we calculate voteshares as the percent of the votes cast for the Democratic candidate, out of all ballots cast, including undervotes. Thus:

$$\text{Democratic Voteshare} = \frac{\text{Democratic Votes}}{\text{Dem. Votes} + \text{Rep. Votes} + \text{Undervotes}} \qquad (1)$$

In the story of suppression caused by accidents, we should expect some voters who intended to vote Democratic instead undervote, thus the numerator gets smaller while the denominator stays the same size as votes are simply transfered from Democratic votes to undervotes. Thus the Democratic return, as a share of *all votes cast* should be decreased by accidents. The Republican voteshare would similarly be reduced. Figure 3.1 plots the voteshares of the Democratic candidate for District 13 vertically and for Senate horizontally. These two measures should predict each other; as more (or fewer) voters in a district cast ballots for the Democratic party in the Senate, we would expect more (or fewer) voters to also cast a ballot for the Democratic candidate for District 13 in the House race[4]. The blue points represent all districts outside Sarasota county, and the red points those inside Sarasota county.

[3] certainly, some of the suppressed voters who did not see the Congressional race on the

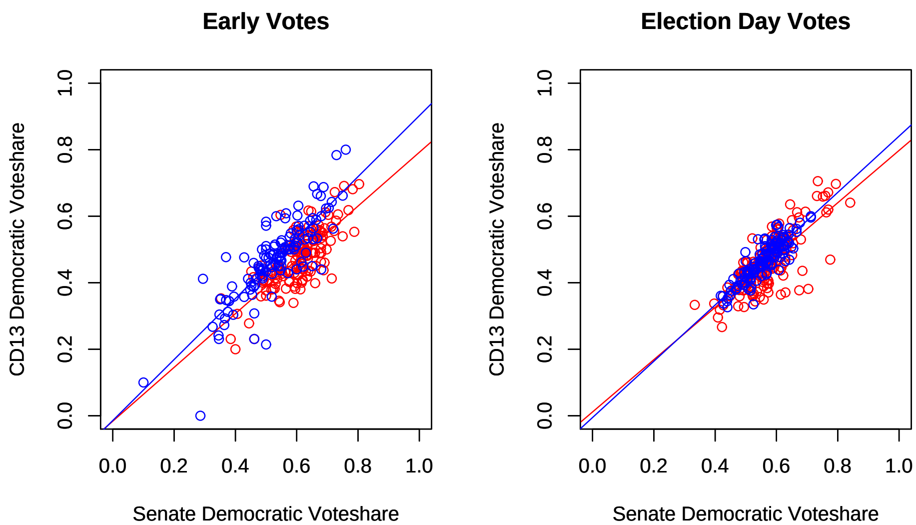

Figure 1: *caption.*

The blue and red lines summarize these relationships, respectively. What can clearly be seen is that at any level of Democratic support for the Senate, the number of Democratic votes for the District 13 race is below what is expected. The red points are clustered horizontally below the blue points and the red summary line dips below the blue summary line[5]. This effect seems to be more pronounced in the early voting than in the election day voting, which agrees with the story that some poll workers were warned of this problem and tried to warn voters who voted on the day of the election. The slope of the lines in Sarasota county is smaller than in the other counties, which is what we would expect if some constant fraction of all votes were being randomly suppressed and converted to undervotes.

It might be argued that this is simply some strange artifact of Sarasota county; perhaps voters in Sarasota are different than elsewhere, and their behavior can not be predicted from the Senate results in the same way as other counties. This can be shown to be false since this difference in Sarasota county is not found in any other statewide race on the ballot. In every other statewide race the relationship predicting voteshare from other election results is exactly the same within Sarasota county as it was in the other counties (These results are presented in the appendix). Moreover, figure 3.1 shows the relationship

ballot might have gone on deliberately undervote in that race if they had seen the choice

[4]The Senate is chosen for these examples as of all races it has the strongest relationship with the Congressional race.

[5]Similarly, the Republican voteshare in Sarasota county falls below what would be expected given the Senate races. The figures are not shown but are as predicted by accidents suppressing Republican votes into undervotes.

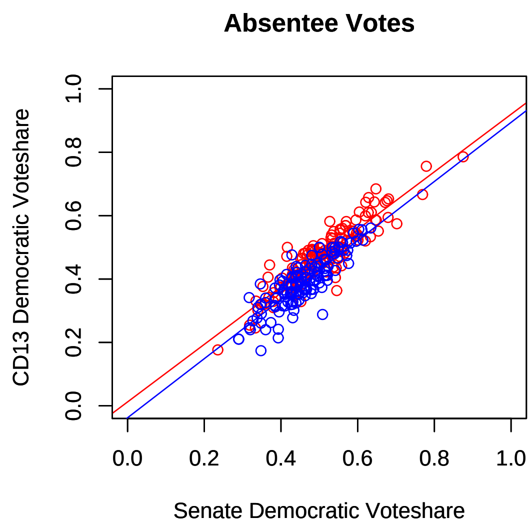

Figure 2: *caption.*

between Senate voting and House voting among absentee ballots. Absentee balloting, which did not suffer the design problem of the touch screen voting, Sarasota county has exactly the same pattern between Senate races and the District 13 race. The red Sarasota line and blue line for other counties now have the same slope, and although the Sarasota line is drawn slightly higher than the blue line, it is not statistically distinguishable.

3.2. *Suppression by Indifference*

There is an alternate stylized story for how the suppressed intended votes may have turned out. We might consider two types of voters, engaged and unengaged. Engaged voters have followed the District 13 election closely, clearly prefer one candidate over another and care deeply about the outcome. When faced with the choice, engaged voters vote for the candidate they strongly prefer. Unengaged voters may have no information about the District election, or are disenchanted with both candidates, or for some other reason do not care about the outcome of the House race. When faced with the choice, unengaged voters may deliberately choose to undervote, that is not pick a candidate, or they might choose between the candidates in some random fashion, out of a perceived duty to vote. Unengaged voters may be very knowledgeable and care deeply about other races on the ballot, we simply assume that they are not interested in the House race.

In these two hypothetical extremes, engaged voters might be expected to search the ballot for their preferred House candidate, and would not be likely to be tripped up by the design flaw. Moreover, if they do initially accidentally miss voting, when the last screen warns them they have not voted in the House election, they are inclined to make the added effort to go back and correct this oversight. Conversely, unengaged voters are not seeking out the House election on the ballot, so are possibly more likely to accidentally miss voting, and when they are warned they have made this error, they are less likely to spend the effort to go back and correct their undervote. In this story, voters who have a distinct preference are less likely to be suppressed, and voters who are indifferent between the candidates are more likely to be suppressed.

If this model were true, at the extreme, if all suppressed votes were of wholly unengaged voters, then the intended votes would have been equally split between the two candidates. Thus if about 14,000 votes were suppressed, each candidate would have received an equal apportionment of about 7,000 each. Jennings would not pick up votes on her opponent, and Buchanan would still have won the election by the same 369 votes.

If we look at the voteshare of the Democratic candidate among only votes cast for the Democratic or Republican candidates, that is excluding undervotes, as:

$$\text{Democratic Voteshare} = \frac{\text{Democratic Votes}}{\text{Dem. Votes} + \text{Rep. Votes}} \tag{2}$$

Then in this story when suppression occurs, an equal number of votes should be removed from the Democratic and Republican Candidate. If the Democratic candidate was previously winning by some number of votes, say 100, then subtracting an equal number of votes from each candidate will cause the voteshare for the winning candidate to *increase* as a 100 vote lead, among a small total number of votes cast will look like a greater ratio. If the previous story from the last section was instead correct, and suppression was caused completely by accident, then votes would be removed from this measure in proportion to how the votes were cast. Thus, under the "accident" story the Democratic (or Republican) voteshare, as calculated above, should not change when suppression occurs, but under the "indifference" story, the winner in any precinct should seem to have a greater share of the cast ballots when indifferent voters are suppressed.

Again, figure 3.2 shows the relationship between the voteshare in the Senate and House races, but now using this different measure of the voteshare that excludes undervoters. Sarasota precincts are again plotted in red and other counties in blue. What we see here is that the Sarasota distribution, and the red line, is above the blue distribution and blue line. Thus the "margin of victory" is more dramatic in Sarasota precincts in the House race than would have been predicted given the relationship between the margin of victory in Senate and House races in the other counties. This supports the theory that suppression did not occur completely randomly, but instead voters who were indifferent were more likely to be suppressed, causing the margin of victory to increase.

3.3. *A Mixture of Factors*

The true process that occurred in Sarasota county is undoubtedly a mixture of these two hypothetical stories. Almost certainly some voters were suppressed by the ballot design completely at random, and completely by accident. These

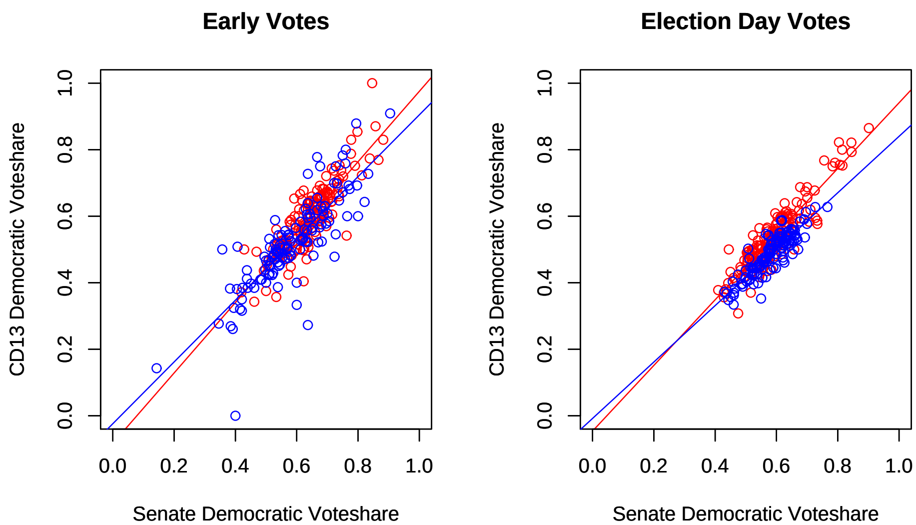

Figure 3: *caption.*

voters would be drawn randomly from the set of voters in the precinct. Also, however, the last warning screen on the electronic ballot that rescued some voters from being suppressed was probably more likely to rescue engaged voters than voters who were unengaged or indifferent in that race and did not want to spend extra effort to vote in a race they had weak or no preference in. Thus some greater proportion of the suppressed vote was less engaged voters who might be expected either to be indifferent, and thus to split their vote evenly between the candidates, or low information voters, who might be more likely to be voting a straight party ticket. It is also possible that the probability of being confused by the electronic ballot is not uniform across all voters, but may likely be correlated with age and other demographics as is true in other setting of ballot error (see for example Tomz and van Houweling, 2003). For all these reasons, the set of voters who were suppressed may not be a completely random sample of the set of voters, and simple reweighting of the observed vote shares is flawed and inaccurate.

4. A COMPOSITIONAL MODEL FOR VOTESHARE REALLOCATION WITH AUXILIARY INFORMATION

Our approach will be to use the available aggregated precinct data to estimate what the precinct vote totals in Sarasota county would have been if the ballot design and equipment had been equivalent to those used in the other counties in this congressional district. In precincts in district thirteen which are outside Sarasota county we can observe all the relationships between the results in other races and the result in the thirteenth Congressional race. These relationships will help us predict how we would have expected the Congressional election in each Sarasota precinct to turn out, given that we know how all the other races turned out in the same precinct.

In precincts inside Sarasota county, we obviously do not observe the counterfactual answer we are seeking, so the Sarasota precincts cannot straightforwardly contribute information to our model of the relationships between the Congressional race and the other races on the ballot. However, these precincts do also contribute some information to our statistical model. Although we do not know how the vote totals would have turned out if there had been no ballot design flaw, we do know that these totals must be higher than the totals that resulted on the day of the election with the flawed ballots. That is, the ballot flaw could only transfer intended votes from each candidate and turn them into undervotes. Therefore, although we can not observed the intended votes, we know each candidate would have received *more* votes than they received on the day of the election. The observed vote totals in Sarasota are clearly in error, and too low, however, they contain some information as they give lower-bounds to each candidates possible vote total. Thus, our statistical model makes two key assumptions:

- *The relationships between electoral races are the same across all precincts in congressional district thirteen.* This is not an assumption that all districts appear or vote the same. Clearly some vote heavily for one candidate, and some for another. Rather, we are assuming the ability to predict one race given knowledge of all the other races, applies inside Sarasota in the same way that it applies in the other precincts in this district.

- *Both candidates, Buchanan and Jennings, would not have received fewer votes if the ballots were correctly designed, than they did on the day with the flawed ballot design.* Succinctly, the error in the ballot design did not add votes to either candidate, but only moved votes into the undervote column.

From these assumptions, we derive a statistical model. Our model assumes the the voteshare for each candidate, as well as the proportion of voters intentionally undervoting are additive-logistic-normally distributed as in the work of Katz and King (1999), and set up a full-information likelihood function using the constraint that candidate voteshares are nondecreasing. We draw predicted values of the Sarasota precinct totals to give us a distribution of imputed values (King et. al. 2001, Schafer 1997), truncating the posterior distribution to obey our censoring constraint (Honaker et. al. 2002).

Our model gives us a probability density over all precinct results, and thus cumulatively, over all election outcomes. We take one thousand random draws from this predicted density, and calculate one thousand predicted election outcomes. From this we can answer the questions raised in the previous sections, such as, what fraction of the undervotes were suppressed votes of voters who intended to cast a ballot for a candidate, how would those votes have broken out between the two candidates, and how might that have influenced the election outcome. Additionally, and crucially, we can express our degree of confidence in each of these quantities.

4.1. *Compositional Transformations*

For the various counties considered here we know how many votes were cast in each precinct for each ballot choice (Republican, Democratic, or Undervote) and by each voting method (early voting, election day voting, and absentee voting).[6] Let $V_{i,c}^{e,m}$ represent the vote share in precinct i, race e, by method $m \in \{\text{early}, \text{e.d.}, \text{abs.}\}$, for choice $c \in \{D, R, U\}$.

For any given race and voting method (that is, temporarily ignoring superscripts), individual vote shares are constrained to the simplex,

$$V_{i,c} \in [0,1] \qquad \forall\ i, c, \tag{3}$$

and the set of votes in a precinct across the three choices sums to unity,

$$V_{i,R} + V_{i,D} + V_{i,U} = 1 \qquad \forall i. \tag{4}$$

The space of each vector V_i is therefore the three dimensional simplex. For compositional data in a J-dimensional simplex, the transformation of Aitchison (1986) creates a set of $J-1$ log ratios each of which compare the vote of one party to that of a baseline or reference party. Without loss of generality we use the Democratic party as our reference choice, and this yields two transformations:

$$Y_{i,RD}^{e,m} = ln\left(\frac{V_{i,R}^{e,m}}{V_{i,D}^{e,m}}\right) \tag{5}$$

$$Y_{i,UD}^{e,m} = ln\left(\frac{V_{i,U}^{e,m}}{V_{i,D}^{e,m}}\right) \tag{6}$$

The set of log vote ratios Y are now individually and collectively unconstrained. Examination of such ratios in other contexts has found them to be well fitted by a multivariate t or multivariate normal distribution (Katz and King 1999, Jackson 2002, Tomz et al. 2002). Thus. our key modeling assumption is that collectively Y_i are joint multivariate normal across all relevant c, e, and m. The reverse transformation from Y to V implies that the vote shares themselves are distributed additive-logistic-normal.

4.2. *Imputation*

Define S as a dichotomous indicator that is one in Sarasota county and zero elsewhere; let V and Y be observed votes shares and transformations as set out above; and, let $\overset{*}{V}$ and $\overset{*}{Y}$ be the latent vote shares and transformations that would have been observed if there were no ballot flaw in that race. Clearly, $\overset{*}{Y}_i^{CD13} = Y_i^{CD13}$, $\forall s_i = 0$, but elsewhere, $\overset{*}{Y}_i^{CD13}$ is unobserved. However, many other races and election methods are observed in Sarasota County and its neighbors.

[6] We discard votes for the minor candidates who contested the gubernatorial race and for all write-ins, and we also discard overvotes. All of the discarded votes are negligible totals in the races studied here.

The twelve election methods that we include in our imputation model are listed in Table 1. The vote shares from each method are transformed into two log vote ratios, the ratio of the Republican to Democratic vote, and the ratio of the undervote to the Democratic party voteshare.[7]. We omit the gubernatorial race because it was the race that shared a ballot screen with the CD 13 race in Sarasota County. Absentee votes for the CD 13 race are included as forecasting variables as these ballots are paper and not subject to any of the proposed mechanisms for the CD 13 ballot failure. We include all precincts in CD 13 from Charlotte, Hardee, Manatee, and Sarasota Counties except the small number of precincts in Manatee that were split between the CD 11 and CD 13 races. As a point of notation, from this point onward we refer to the log vote ratios in the early and election day voting in the CD 13 race as Y's and the log ratios of all these other races in Table 1 as X's although the latter are still constructed by equation 5.

If we consider $\overset{*}{Y} = Y$ for all $s_i = 0$ and all observations of $\overset{*}{Y} = Y$ within Sarasota as completely missing data (with observed covariates X), then our model has the same architecture as any conventional multivariate normal imputation model (Rubin 1986, Schafer 1997, King et al. 2001) We can estimate the posterior distribution of the missing values and draw imputations from this distribution to create fully observed datasets from which it is straightforward to create quantities of interest such as the vote totals of each candidate.

However, as there are only two patterns of missingness in our data, the critical complication of imputation algorithms, running large numbers of simultaneous equations, can be avoided.[8] Ignoring covariance between early and election day returns, within the CD 13 race the imputation model is simply two sets of bivariate normal regressions:

$$\begin{aligned} (\overset{*}{Y}_{RD}, \overset{*}{Y}_{UD}) &\sim f_{\text{bivariate normal}}(\mu_{RD}, \mu_{UD}, \Sigma) \\ \mu_{RD} &= \mathbf{X}\beta \\ \mu_{UD} &= \mathbf{X}\gamma \end{aligned} \tag{7}$$

where $\mathbf{X}$ consists of 14 log vote ratios from the elections in Table 1 plus a constant vector. Imputations of $\overset{*}{Y}$ from this model yields completely observed data. However, given the simplicity of the patterns of missingness in our model, we can elaborate on the conventional multivariate normal model to include vote shares within Sarasota as censored observations.

4.3. *Constraints*

Observed CD 13 vote shares in Sarasota County contain some information about latent values. If the Sarasota ballot had been equivalent to the ballots used in other CD 13 counties, then it is reasonable to assume that some Sarasota

[7]In the judicial retention and amendment returns, the variables are the log ratio of no to yes votes, and the log ratio of undervotes to yes votes. Early returns in CD 13 are predicted with early returns from the other elections, and the two Absentee variables. Election day returns are predicted with election day and absentee variables.

[8]All observations are either fully observed or are missing the four log ratios of early and election day voting in the CD 13 race.

ballots which contain CD 13 undervotes would have registered a vote for a candidate while no votes successfully cast for either candidate would change. Thus, in Sarasota County the vote shares for both Buchanan and Jennings must be strictly increasing in undervotes allocation and the vote share of undervotes correspondingly decreasing. Moreover, there are upper bounds for how much Buchanan and Jennings vote shares could change if all Sarasota CD 13 undervotes were allocated to these two candidates. In the limit, all undervotes could break for Buchanan or Jennings. Thus, the observed vote shares in Sarasota give us a series of bounds on the latent vote shares:

$$\overset{*}{V}_{i,U} \leq V_{i,U} \qquad \forall i : s_i = 1 \tag{8}$$

$$(V_{i,R} + V_{i,U}) \geq \overset{*}{V}_{i,R} \geq V_{i,R} \qquad \forall i : s_i = 1 \tag{9}$$

$$(V_{i,R} + V_{i,U}) \geq \overset{*}{V}_{i,D} \geq V_{i,D} \qquad \forall i : s_i = 1 \tag{10}$$

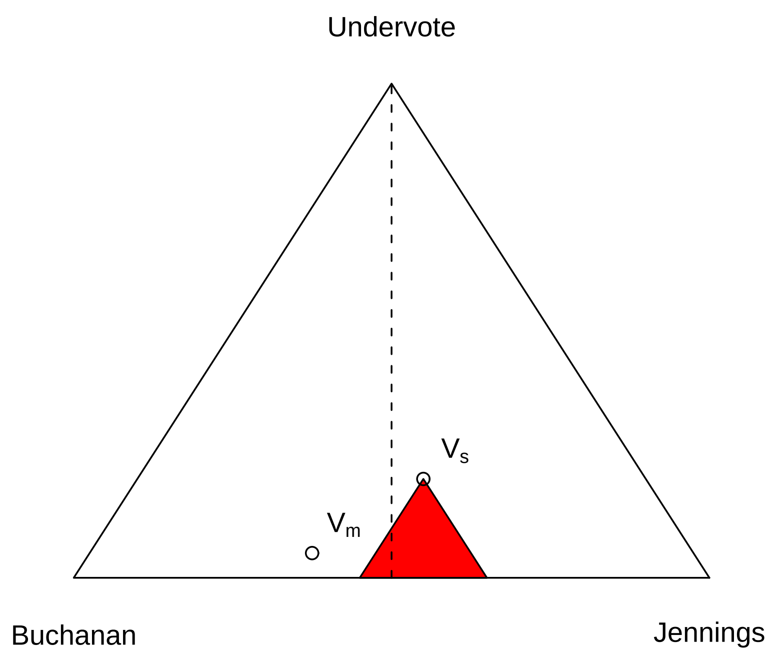

Figure 4: Ternary plot of Hypothetical Sarasota and Manatee County Precincts

Note: The two precincts are denoted V_m (Manatee County) and V_s (Sarasota County). In V_s, which suffers from the ballot flaw described in the body of this paper, the latent vote share must be closer to the Buchanan and Jennings corners and thus can only fall in the shaded region. The vertical dashed line defines which candidate won the plurality.

As an illustration, imagine reported results in two hypothetical precincts, V_s in Sarasota County and V_m in neighboring Manatee County. In Manatee, where there was no ballot format problem, $\overset{*}{V}_m = V_m$. In the Sarasota precinct, assume

Jennings received 45 percent of the vote and Buchanan 35 percent, and assume that the remaining 20 percent of ballots were CD 13 undervotes. In the ternary plot in Figure 4, this precinct is represented by the point V_s. The shaded region represents all points that are closer to both bottom vertices than V_s is, thus the set of possible election results if the intended votes were counted. This shaded region is the space where $\overset{*}{V}_s$ might be located.[9] These bounds on $\overset{*}{V}$ imply a set of bounds on $\overset{*}{Y}$, the most straightforward of which is:

$$\overset{*}{Y}_{i,UD} \leq Y_{i,UD} \tag{11}$$

Additionally, if we knew the true undervote, $\overset{*}{V}_{i,U}$, we could define the functions:

$$\begin{aligned} Y^+_{i,RD}(\overset{*}{V}_{i,U}) &= ln\left(\frac{V_{i,R} + (V_{i,U} - \overset{*}{V}_{i,U})}{V_{i,D}}\right) && (12) \\ Y^-_{i,RD}(\overset{*}{V}_{i,U}) &= ln\left(\frac{V_{i,R}}{V_{i,D} + (V_{i,U} - \overset{*}{V}_{i,U})}\right), && (13) \end{aligned}$$

which provide bounds on

$$Y^+_{i,RD}(\overset{*}{V}_{i,U}) \geq \overset{*}{Y}_{i,RD} \geq Y^-_{i,RD}(\overset{*}{V}_{i,U}). \tag{14}$$

We simplify these functions to their limiting values as $\overset{*}{V}_{i,U} \rightarrow 0$

$$\begin{aligned} Y^+_{i,RD} &= ln\left(\frac{V_{i,R} + V_{i,U}}{V_{i,D}}\right) && (15) \\ Y^-_{i,RD} &= ln\left(\frac{V_{i,R}}{V_{i,D} + V_{i,U}}\right) && (16) \end{aligned}$$

Using equation 11 and the simplified form of equation 14 we can set the limits of integration for:

$$L(\beta, \gamma, \Sigma | S_i = 1) = \int_{Y^-_{i,RD}}^{Y^+_{i,RD}} \int_{-\infty}^{Y_{i,UD}} p_{\text{bvn}}(r, s | \mathbf{X}_i\beta, \mathbf{X}_i\gamma, \Sigma) \delta s \; \delta r \tag{17}$$

while the precincts outside Sarasota more straightforwardly contribute:

$$L(\beta, \gamma, \Sigma | S_i = 0) = p_{\text{bvn}}(Y_{i,RD}, Y_{i,UD} | \mathbf{X}_i\beta, \mathbf{X}_i\gamma, \Sigma) \tag{18}$$

[9] The vertical dashed line separates points closer to Buchanan from points closer to Jennings. Neither candidate has received a majority in this precinct, but Jennings has a plurality. Buchanan could still win the precinct plurality if enough undervotes fell his way, as the shaded region crosses this line.

4.4. *Rejection Sampling*

We parametrically bootstrap the parameters from our imputation model. From each bootstrapped set of parameters we create one imputed dataset where all the election outcomes are the same as observed values except that early and election day CD 13 vote shares in Sarasota County precincts are draws from their posterior distributions, conditional on other observed elections in those precincts. Although five or ten imputed datasets is sufficient in most analyses, we want to create confidence intervals of some quantities and so impute 1,000 datasets.

Our imputation model as previously discussed is multivariate normal in the space of the Ys. The quantities of interest to us, however, are the vote totals for each candidate. This requires transforming imputed log vote ratios back to vote shares and then multiplying these vote shares by the total turnout in each precinct. The reverse transformations are:

$$\widetilde{V}_{i,U} = \exp(\widetilde{Y}_{i,UD})/W_i, \quad \widetilde{V}_{i,D} = 1/W_i, \quad \widetilde{V}_{i,R} = \exp(\widetilde{Y}_{i,RD})/W_i, \tag{19}$$

$$\text{where} \quad W_i = 1 + \exp(\widetilde{Y}_{i,RD}) + \exp(\widetilde{Y}_{i,RD}). \tag{20}$$

Our imputed values are drawn from an untruncated conditional posterior yet we want values conditional on V. Therefore we need draws from a truncated distribution that obeys $\widetilde{V}_{i,U} \leq V_{i,U}$, $\widetilde{V}_{i,D} \geq V_{i,D}$, and $\widetilde{V}_{i,R} \geq V_{i,R}$. Following Honaker, Katz and King (2002), we rejection sample each vector $\widetilde{V}_i$ until every observation passes all constraints. This rejection sampling needs to be done on the imputations regardless of whether the CD 13 returns in Sarasota County are treated as entirely missing or censored values.

5. ESTIMATION ACROSS RACES WITH UNDERVOTING

Individual models were estimated for early voting (that is votes entered on the touch screen before the day of the general election) and election day voting, as the certified returns were issued separately, and these might represent different types of voters. The set of races used as predictive variables are described in table .

5.1. *Florida's 13th Congressional Election*

The results of our analysis for the 13th Congressional race are presented in table 2. Key here is how many more votes Buchanan and Jennings would have received if Sarasota undervotes had not been influenced by ballot design. This is labeled *pickup* for each candidate. The undervote estimates are negative because the model supports the belief that most undervotes were intended to have been cast for a candidate. The undervote pickup could not be positive because that is the assumption of the use of the auxiliary bounding information. Looking at the early voting, we see that we are 95 percent confident that there would have been between 4750 and 4401 fewer early voting undervotes if there had been no ballot design problem in Sarasota county. Similarly we are 95 percent confident there were between 9509 and 10165 undervotes caused by voting technology on the day

Race	Methods
Congressional District 13	Absentee
U.S. Senate	Early, Election Day, Absentee
Agricultural Commissioner	Early, Election Day
Chief Financial Officer	Early, Election Day
Lewis Supreme Court Retention	Early, Election Day
Amendment 8	Early, Election Day

Table 1: Races and Voting Methods used in Imputation Model

Note: The Sarasota County absentee ballots in the CD 13 race can be used to forecast early and election day totals as the former did not suffer from any of the touchscreen formatting problems.

of the election. The rate of mistaken undervoting as a fraction of all undervotes decreases in the estimates for the day of the election, and this is consistent with the story that some poll workers attempted to correct this problem on the day of the election. In all, we are 95 percent confident that between 14040 and 14792 voters in Sarasota county intended to cast a valid vote in the Congressional race but did not. Of these votes reallocated by the model, we find that they would have broken substantially in the direction of Jennings. We estimate Jennings should have picked up 8018 more votes and Buchanan would have picked up 6404. Across all the imputations, the distribution of the difference between the pickup of Jennings and the pickup of Buchanan is calculated. Whenever this value is greater than 369 then the model predicts that if Sarasota had not suffered a ballot problem, then Jennings would have received enough of the undervotes to overturn the eventual margin of victory, and reverse the outcome of the election. The area to the right of 369 is the probability of this event, which we calculate as 98.4 percent.

We can compare these results to a logistic analysis conducted with individual level "ballot image" data. This procedure (and the results below) are detailed in our earlier work (Frisina et al. 2008). To estimate the probability that a undervote would have been cast for a candidate under a different ballot technology we estimate a series of separate models. Using data from surrounding counties without the ballot error, we estimate the probability a voter would have voted, conditional on their votes choices in the races in table 5. To estimate how Sarasota CD 13 undervotes that should have been valid votes would have been divided between Buchanan and Jennings we assume that, conditional on votes in other races on the ballot, those Sarasota voters undervoting in CD13 were no more or less likely to support Buchanan or Jennings than those who did not undervote in this race. By this we can estimate the probability that each Sarasota undervoter would continue to undervote if using the Charlotte ballot, and the probability of voting for Buchanan or Jennings conditional on voting at all. Multiplying these probabilities across all undervoters and summing gives us our estimate of the reallocation provided in table 3. Confidence intervals are obtained via a bootstrap.

As compared to the imputation model from the precinct aggregated data,

Table 2: Summary of Allocation Results from Compositional Data using Multiple Imputation
13th Congressional Race, Sarasota County

	Jennings	Buchanan	Undervote
EARLY VOTING:			
	Jennings	Buchanan	Undervote
Estimated Vote:	16944	12939	830
Estimated Pickup:	2477	2106	-4582
Lower bound of 95% Confidence Interval:	2159	1811	-4750
Upper bound of 95% Confidence Interval:	2779	2400	-4401
ELECTION DAY VOTING:			
	Jennings	Buchanan	Undervote
Est. Vote:	45387	40774	2511
Est. Pickup:	5541	4298	-9840
Lower bound of 95% Confidence Interval:	5019	3837	-10165
Upper bound of 95% Confidence Interval:	6064	4811	-9509
EARLY + ELECTION DAY VOTING:			
	Jennings	Buchanan	Undervote
Election Returns:	54313	47309	17763
Estimated Totals:	62331	53713	3341
Projected Pickup:	8018	6404	-14422
Lower bound of 95% Confidence Interval:	7377	5853	-14792
Upper bound of 95% Confidence Interval:	8612	6969	-14040
Predicted probability of Jennings's pickup > 369: 0.984			

Table 3: Summary of 13th Congressional Race Allocation Results from Ballot Image Data with Logistic Regression

	Jennings	Buchanan	Undervote
Election returns	54313	47567	17825
Estimated Vote:	63261	53420	3024
Estimated Pickup:	8948	5853	-14801
Lower bound of 95% Confidence Interval:	8710	5635	-14347
Upper bound of 95% Confidence Interval:	9162	6059	-15266
Predicted probability of Jennings's pickup > 369: ≈ 1			

here we find stronger evidence that Jennings would have won the CD 13 election if Sarasota had used the same machines as were used in Charlotte county. The model predicts about 80 percent more votes would be picked up by Jennings than in the precinct level analysis. Key to our comparison of the methods, however, the precinct-level confidence interval was roughly plus or minus 1111 votes. Here the confidence interval is only plus or minus 221 votes. Thus, through the use of ballot-level data, we improved the precision of the our estimates five-fold.

5.2. *Florida's Attorney General Race*

Summary of Allocation Results from Compositional Data using Multiple Imputation
Attorney General's Race, Charlotte County

EARLY VOTING:			
	Campbell	McCollum	Undervote
Estimated Vote:	8180	8173	739
Estimated Pickup:	2020	1413	-3434
CI95 lower:	1796	1183	-3538
CI95 upper:	2240	1633	-3327
ELECTION DAY VOTING:			
	Campbell	McCollum	Undervote
Estimated Vote:	12789	14703	1400
Estimated Pickup:	3468	2324	-5792
CI95 lower:	3215	2078	-5955
CI95 upper:	3717	2583	-5620
EARLY + ELECTION DAY VOTING:			
	Campbell	McCollum	Undervote
Election Returns:	15481	19139	11365
Estimated Totals:	20969	22876	2140
Projected Pickup:	5488	3737	-9225
CI95 lower:	5140	3388	-9419
CI95 upper:	5808	4094	-9020

The results for the race for Attorney General from the imputation model are presented below. This model reallocates undervoters in Charlotte county who suffered from a similar ballot design problem as Sarasota county, with the two candidate Attorney General race sharing a ballot page below the fierce and crowded seven candidate Governor's race. Undervoting in this race in Charlotte did not receive the same media attention as the Sarasota case because the race for Attorney General was statewide, and won by McCollum by a large margin. The estimated relative magnitude of the undervoting problem is about the same as in the Sarasota case. The model predicts that about 81 percent of recorded undervotes would have been cast for a candidate if the race had been given its own ballot page, as in the surrounding counties. Interestingly, these undervotes break significantly for Campbell, who is the losing candidate in Charlotte county. Campbell is once more the Democratic candidate in this race, as Jennings was in Sarasota, so if this is correct, this undervoting problem did not simply effect the most popular candidate in the county where the undervote occurred, but may have predominantly effected Democratic candidates. This may be a result of Democratic voters being lower information or in demographic groups more likely to be confused by technology, or this may be a result of Democratic voters being more likely to vote straight party tickets and searching for one Democratic candidate on every ballot page. The logistic analysis of this race with the individual level "ballot image" data is not complete at this time, but will make for an authoritative check of this intriguing result.

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